D.A.V. INSTITUTIONS, CHHATTISGARH

PRACTICE PAPER-6 : 2023-24

CLASS – XII

SUBJECT- MATHEMATICS (041)

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

1. **All questions are compulsory.**
2. **The question paper has five sections. Section–A, Section-B, Section-C, Section-D and Section–E. There are 38 questions in the question paper.**
3. **Section–A has 18 MCQ questions and 2 Assertion- Reason based question of 1 marks each. Section–B has 5 Very Short Answer (VSA) type questions of 2 marks each, Section-C has 6 Short Answer (SA) type questions of 3 marks each, Section–D has 4 Long Answer (LA) type questions of 5 marks each and Section–E has 3 case based questions of 4 marks each.**
4. **There is no overall choice. However internal choice have been provided in some questions. Attempt only one of the alternatives in such questions.**
5. **Wherever necessary, neat and properly labelled diagram should be drawn.**

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The vector of the direction of the vector $\hat{i}-2j+2k$ that has magnitude 9 is

(a) $\hat{i}-2j+2k$ (b) $\frac{\hat{i}-2j+2k}{3}$ (c) $3(\hat{i}-2j+2k)$ (d) $9(\hat{i}-2j+2k)$

2. If P(A) = 2/5, P(B)= 3/10 and P(A ∩ B) = 1/5, then P(A'/B').P(B'/A') is equal to

(a) 25/42 (b) 5/6 (c) 5/7 (d) 1

3. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/4. What is the probability that the student knows the answer given that he answered it correctly?

(a) 11/13 (b) 7/13 (c) 12/13 (d) 9/13

4. If the position vector $\vec{a}$ of the point (5, n) is such that $|\vec{a}| = 13$, then the value(s) of n can be

(a) ±12 (b) ±8 (c) Only 12 (d) Only 8

5. $\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx = ?$

(a) $\frac{-e^x}{1+x^2}+C$ (b) $\frac{e^x}{1+x^2}+C$ (c) $\frac{-e^x}{(1+x^2)^2}+C$ (d) $\frac{e^x}{(1+x^2)^2}+C$

6. Two-line $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the x-y plane has coordinates

(a) (2, 4, 7) (b) (-2, 4, 7) (c) (2, -4, -7) (d) (2, -4, 7)

7. $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx = ?$

(a) $(x+1)e^{x+\frac{1}{x}}+C$ (b) $xe^{x+\frac{1}{x}}+C$

(c) $-xe^{x+\frac{1}{x}}+C$ (d) $(x-1)e^{x+\frac{1}{x}}$

8. The area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is

(a) $20\pi^2$ sq. units (b) 25π sq. units (c) 20π sq. units (d) $16\pi^2$ sq. units

9. If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ, then the value of $|\vec{a}-\vec{b}|$ is

(a) $2\cos\frac{\theta}{2}$ (b) $2\sin\frac{\theta}{2}$ (c) $2\cos\theta$ (d) $2\sin\theta$

10. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to

(a) $4\pi^2$ sq units (b) 4π sq units (c) 2π sq units (d) $2\sqrt{2}\pi$ sq units

11. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=x\sin\left(\frac{dy}{dx}\right)$ is

(a) 1 (b) 2 (c) 3 (d) not defined

12. $\int \sin^3(2x+1)dx=?$

(a) $\frac{1}{2}\cos(2x+1)+\frac{1}{3}\cos^3(2x+1)+C$ (b) $-\frac{1}{2}\cos(2x+1)+\frac{1}{6}\cos^3(2x+1)+C$

(c) $\frac{1}{8}\sin^4(2x+1)+C$ (d) none of these

13. If A and B are invertible matrices, then which of the following is not correct.

(a) adj $A = |A|. A^{-1}$ (b) det $(A^{-1}) = [\det(A)]^{-1}$

(c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A + B)^{-1} = B^{-1} + A^{-1}$

14. Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when

(a) $x > 2$ (b) $1 < x < 2$ (c) $x = 2$ (d) $x > 3$

15. If A is a square matrix of order 3 and $|A| = -5$, then |adj A| is:

(a) 125 (b) –25 (c) 25 (d) ±25

16. If $A = \begin{bmatrix} 2x & 6 \\ -1 & 1 \end{bmatrix}$ is a singular matrix, then x :

(a) 3 (b) – 3 (c) 1 (d) – 2

17. The integrating factor of the differential equation $\frac{dy}{dx}(x\log x)+y=2\log x$ is

(a) e^x (b) log x (c) log(log x) (d) x

18. The value of $\tan^2(\sec^{-1}2) + \cot^2(\mathrm{cosec}^{-1}3)$ is

(a) 5 (b) 11 (c) 13 (d) 15

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): If manufacturer can sell x items at a price of Rs. $\left(5-\frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5}+500\right)$. Then, the number of items he should sell to earn maximum profit is 240 items.

Reason (R): The profit for selling x items is given by $\frac{24}{5}x-\frac{x^2}{100}-300$.

20. Assertion(A): Determinant of a skew-symmetric matrix of order 3 is zero.
Reason(R): For any matrix A, |A'| = |A| and |-A| = |A|.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$

22. Find the values of x and y from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix}+\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

OR

If $A=\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

23. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then find the probability that both drawn balls are black.

24. Solve $(1 + x^2) \sec^2 y dy + 2x \tan y dx = 0$, given that $y = \pi/4$ when $x = 1$.

25. Find the area of parallelogram whose adjacent sides are represented by the vectors $\vec{a}=2\hat{i}-j+k$ and $\vec{b}=3\hat{i}-j$.

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. Find the relationship between a and b so that the function f defined by $f(x)=\begin{cases} ax+1, & if \quad x\le 3 \\ bx+3, & if \quad x>3 \end{cases}$ is continuous at $x = 3$.

27. Solve: $\frac{dy}{dx}=(x+y+1)^2$

OR

Find the general solution of the differential equation: $xdy-ydx=\sqrt{x^2+y^2}dx$

28. Evaluate: $\int \frac{x}{\sqrt{1-x^2+x^4}}dx$

OR

Evaluate $\int_0^2 f(x)dx$, if $f(x)=\begin{cases} -(x-3), x<2 \\ (x-3), x>2 \end{cases}$

29. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$

30. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2j+2k$ and $\vec{b}=\hat{i}+2j-2k$

OR

Using vectors, prove that the points (2, –1, 3), (3, –5, 1) and (–1, 11, 9) are collinear.

31. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$.

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Minimize and maximize $Z = 5x + 2y$ subject to the following constraints:
$x - 2y \leq 2$, $3x + 2y \leq 12$, $-3x + 2y \leq 3$, $x \geq 0$, $y \geq 0$

33. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by $R = \{(a, b): |a - b|$ is a multiple of 4$\}$ is an equivalence relation. Find the set of all elements related to 1.

OR

Show that the function $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \dfrac{x}{1+|x|}, x \in R$ is one-one and onto function.

34. Find the vector equation of the line passing through (1, 2, – 4) and perpendicular to the two lines: $\dfrac{x-8}{3} = \dfrac{y+19}{-16} = \dfrac{z-10}{7}$ and $\dfrac{x-15}{3} = \dfrac{y-29}{8} = \dfrac{z-5}{-5}$

OR

Find the shortest distance between the lines whose vector equations are:
$\vec{r} = (\hat{i} + j) + \lambda(2\hat{i} - j + k)$ and $\vec{r} = (2\hat{i} + j - k) + \mu(3\hat{i} - 5j + 2k)$

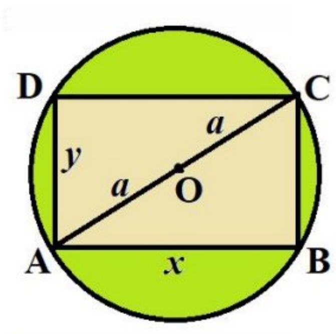
35. Evaluate: $\displaystyle\int \frac{x^2}{x^4 - x^2 - 12}dx$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case-Study 1:

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Find the perimeter of rectangle in terms of any one side and radius of circle.
(ii) Find critical points to maximize the perimeter of rectangle?
(iii) Check for maximum or minimum value of perimeter at critical point.

OR

(iii) If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region.

37. Case-Study 2:

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.

(i) Write the matrix summarizing sales data of 2019 and 2020.
(ii) Find the matrix summarizing sales data of 2020.
(iii) Find the total number of cars sold in two given years, by each dealer?

OR

(iii) If each dealer receives a profit of = 50000 on sale of a Hatchback, 100000 on sale of a Sedan and 2200000 on sale of an SUV, then find the amount of profit received in the year 2020 by each dealer.

38. Case-Study 3:

In a school, teacher asks a question to three students Kanabh, Raj and Shubi respectively. The probability of solving the question by Kanabh, Raj and Shubi are 40%, 15% and 50% respectively. The probability of making error by Kanabh, Raj and Shubi are 1.5%, 2% and 2.5%.

Based on the given information, answer the following questions:
(i) Find the probability that Shubi solved the question and committed an error.
(ii) Find the total probability of committing an error is solving the question.