ANNEXURE – C

DAV PUBLIC SCHOOLS, ODISHA ZONE

NAME OF THE EXAM: HALFYEARLY, SUBJECT :MATHEMATICS,
CLASS : STD - XII

MARKING SCHEME SET-I

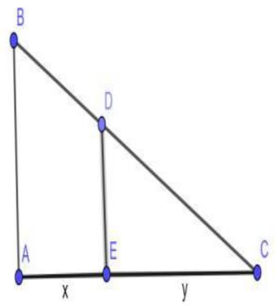
QSTN NO	VALUE POINTS	MARKS ALLOTTED	PAGE NO. OF NCERT TEXT BOOK
	SECTION – A		
1	(a) Reflexive	1 Mark	NCERT
2	(d) 40	1 Mark	Exemplar
3	$(d) -\frac{\pi}{8}$	1 Mark	Exemplar
4	(a) $\frac{-5\pi}{12}$	1 Mark	Exemplar
5	(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1 Mark	NCERT
6	(c)2	1 Mark	NCERT
7	(c) $(cosx + e^x)^{-1}$	1 Mark	NCERT
8	(d) (0,2)	1 Mark	NCERT
9	(c)$10\sqrt{3}cm^2 / \sec$	1 Mark	NCERT
10	(c) 1	1 Mark	Exemplar
11	(c) x + c	1 Mark	Exemplar
12	(d)$\frac{1}{3}\sin^{-1}\frac{3x}{4} + c$	1 Mark	Exemplar

13	b) $\frac{\pi x}{2}-\frac{x^2}{2}+c$	1 Mark	NCERT
14	(a) 2 sq units	1 Mark	Exemplar
15	(b) $\frac{256}{3}$ sq units	1 Mark	Exemplar
16	(c) y	1 Mark	Exemplar
17	(a) 4	1 Mark	Exemplar
18	(a) $e^x + e^{-y} = c$	1 Mark	NCERT
19	(d)	1 Mark	NCERT
20	(b)	1 Mark	NCERT
SECTION – B			
21	For correct one-one proof $y=\frac{2x}{5x+3} \Rightarrow x=\frac{3y}{2-5y}$ For every $y \in R-\left\{\frac{2}{5}\right\}$, there exists $x \in R-\left\{\frac{-3}{5}\right\}$ such that $f(x)=f\left(\frac{3y}{2-5y}\right)=2\left(\frac{3y}{2-5y}\right)\div\left(5\frac{3y}{2-5y}+3\right)=y$ So, f is onto. OR As f(2) = f($\frac{1}{2}$) = $\frac{2}{5}$ So , f is not one – one Let f(x) =1 $\Rightarrow \frac{x}{1+x^2}=1$ $\Rightarrow x^2$-x +1 = 0 $\Rightarrow x \notin R$ *so* , f is not on to.	1 1 1 1	Exemplar

<table>
<tr><td>22</td><td>$$\sin^{-1}\left[\cos\left(8\pi+\frac{3\pi}{5}\right)\right]=\sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$
$$=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)\right]=-\frac{\pi}{10}$$</td><td>1

1</td><td>NCERT</td></tr>
<tr><td>23</td><td>$\tan^{-1}\left(\frac{cosx}{1-sinx}\right)=\tan^{-1}\left(\frac{\cos^2 x/2-\sin^2 x/2}{(cosx/2-sinx/2)^2}\right)$
$=\tan^{-1}\left(\frac{1+tanx/2}{1-tanx/2}\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$
$$=\frac{\pi}{4}+\frac{x}{2}$$</td><td>1 Mark

1 Mark</td><td>Exemplar</td></tr>
<tr><td>24</td><td>Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and



the length of his shadow EC be y m.

Using similarity of triangles, we have $\frac{4}{1.6}=\frac{x+y}{y}$
$$\Rightarrow 3y = 2x$$
Differentiating both sides w.r.t t,we get $3\frac{dy}{dt}=2\frac{dx}{dt}$
$$\frac{dy}{dt}=\frac{2}{3}\times 0.3=0.2$$
At any time t seconds, the tip of his shadow is at a distance of$(x+y)$ m from AB.

The rate at which the tip of his shadow moving

$=\left(\frac{dy}{dt}+\frac{dx}{dt}\right)$ m/s=0.5 m/s

OR

$\frac{dQ}{dx}=0.024x^2-0.1x+20$

∴ Marginal cost at x=2 is 20.296</td><td>1 Mark

1 Mark

1

1</td><td>NCERT</td></tr>
<tr><td>25</td><td>$\int \frac{x-3}{(x-1)^3}e^x dx=\int \frac{x-1-2}{(x-1)^3}e^x dx$

$=\int\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}e^x dx$

$=\frac{e^x}{(x-1)^2}+c$</td><td>1

1</td><td>NCERT</td></tr>
</table>

SECTION – C			
26	$A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$. Put it in the given equation and it is satisfied. $A^2 - 4A + I = O \Rightarrow I = 4A - A^2$. Multiply A^{-1} in both the sides. $\Rightarrow A^{-1}I = 4A^{-1}A - A^{-1}A.A \Rightarrow$ $A^{-1} = 4I - IA \Rightarrow A^{-1} = 4I - A \Rightarrow A^{-1} = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} =$ $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$.	1.5 1.5	NCERT
27	y=$(logx)^x$+$(x)^{cosx}$ y=u+v u=$(logx)^x$ and v=$(x)^{cosx}$ finding $\frac{du}{dx}$ finding $\frac{dv}{dx}$ $\frac{dy}{dx}$=$\frac{du}{dx} + \frac{dv}{dx}$ OR $\begin{cases} \frac{x-4}{\lvert x-4\rvert} + a, if x < 4 \\ a + b, \ if x = 4 \\ \frac{x-4}{\lvert x-4\rvert} + b, if x > 4 \end{cases}$ is a continuous function at x=4. LHL = $\lim_{4^-} f(x)$=$\lim_{4^-}[\frac{(x-4)}{-(x-4)} + a]$ =$\lim_{4^-}[-1 + a]$= a-1 RHL= $\lim_{4^+}[\frac{(x-4)}{\lvert x-4\rvert} + b]$ =$\lim_{4^+}[\frac{(x-4)}{(x-4)} + b]$ =$\lim_{4^+}[1 + b]$=1+b f(4)=a+b As *f is continuous at x=4* *LHL=RHL =f(4)* *a-1 = a+b=1+b* *a-1=a+b&a+b=1+b* *b=-1 &a =1* *So a=1,b=-1*	 0.5 1 1 0.5 1 1 1	NCERT Exemplar
28	$f(x) = 20 - 9x - 6x^2 - x^3$ $\Rightarrow f'(x) = -9 - 12x - 3x^2 = -3(x+1)(x+3)$ $f'(x) = 0 \Rightarrow x = -1, -3$ So f is strictly decreasing in $(-\infty, -3) \cup (-1, \infty)$ and increasing in	1 Mark 1 Mark 1 Mark	NCERT

	(-3, -1).		
29	I= $\int\{\log(\log x) + \frac{1}{(\log x)^2}\}$ dx Let log x = t ⇒x = e^t ⇒ dx = e^t dt ⇒ I = $\int e^t\left(\log t + \frac{1}{t^2}\right)dt$ = $\int e^t\left(\log t + \frac{1}{t} - \frac{1}{t} + \frac{1}{t^2}\right)dt$ =$e^t\left(\log t - \frac{1}{t}\right)$+c = $x(\log(\log x) - \frac{1}{\text{logx}})$ +c	 1 1 1	NCERT
30	I= $\int_0^{\frac{\pi}{4}} \log(1 + tanx)$dx. I = $\int_0^{\frac{\pi}{4}} \log(1 + \tan(\frac{\pi}{4} - x)$dx. I=$\int_0^{\frac{\pi}{4}} \log 2$ dx.-$\int_0^{\frac{\pi}{4}} \log(1 + tanx)$dx. 2I= $\frac{\pi}{4}$ log2 I=$\frac{\pi}{8}$ log2	 1 1 1	NCERT

31	$\frac{dy}{dx} = e^{3x}e^{4y} \Rightarrow \int e^{-4y}dy = \int e^{3x}dx$ $\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3}$ + c, putting x = 0 and y = 0 we get c = -7/12 Hence solution is $\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$ that is $4e^{3x} + 3e^{-4y} - 7 = 0$ OR $y\,dx - (x + 2y^2)\,dy = 0$ $\Rightarrow y\,dx = (x + 2y^2)\,dy$ $\Rightarrow \frac{dx}{dy} = \frac{(x+2y^2)}{y}$ $\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y$ *This is a linear differential equation of the type* $\frac{dx}{dy} + P1x = Q1$ Where P1=$\frac{-1}{y}$ and Q1 = $2y$ Therefore IF = $e^{\int \frac{-1}{y}dy} = e^{-logy} = \frac{1}{y}$ Hence the solution of the given differential equation is $$x\,(IF) = \int Q1(IF)dy \quad + C$$ $\Rightarrow x\frac{1}{y} = \int 2y \times \frac{1}{y}dy + C$ $\Rightarrow\ x\ = 2y^2 + Cy$	1 1 1 0.5 1 0.5 1	NCERT NCERT
SECTION – D			
32	$a, b \in N$ ⇒ab(a+b)=ba(a+b) ∴ (a, b)R(a, b) for all (a, b) ∈ N × N Hence, R is reflexive. Let (a, b), (c, d) be an arbitrary element of N × N such that (a, b)R(c, d). ∴ ad(b+c)=bc(a+d) ⇒cb(d+a)=da(c+b) ⇒(c,d)R(a,b)	1 1.5	NCERT

	$\therefore (a, b)R(c, d) \Rightarrow (c, d)R(a, b)$ for all $(a, b), (c, d) \in N \times N$ Hence, R is symmetric. $ad(b+c)=bc(a+d)$ Also, $cf(d+e)=de(c+f)$ $\Rightarrow adb+adc=abc+bcd$ $\Rightarrow cfd+cfe=dec+def$ $\Rightarrow cd(a-b)=ab(c-d)$ $\Rightarrow cd(f-e)=ef(d-c)$ ....($\Rightarrow aef-bef=-abf+aeb$ $\Rightarrow aef+abf=aeb+bef$ $\Rightarrow af(b+e)=be(a+f)$ $\Rightarrow (a, b)R(e, f)$ $\therefore (a, b)R(c, d)$ and $(c, d)R(e, f) \Rightarrow (a, b)R(e, f)$ for all $(a, b), (c, d), (e, f) \in N \times N$ Hence, R is transitive. Thus, R being reflexive, symmetric and transitive, is an equivalence relation on $N \times N$. OR Here, function $f: R^+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ One-one function: Let $x_1, x_2 \in R^+$ such that $f(x_1) = f(x_2)$ Then, $9x_12 + 6x_1 - 5 = 9x_2 + 6x_2 - 5$ $\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$ $\Rightarrow 9(x_1 + x_2)(x_1 - x_2) + 6(x_1 - x_2) = 0$ $\Rightarrow (x_1 - x_2)[9(x_1 + x_2) + 6] = 0$ $\Rightarrow x_1 - x_2 = 0$ [$\because x_1, x_2 \in R_+ \therefore 9(x_1 + x_2 + 6 \neq 0)$ $\Rightarrow x_1 = x_2, \forall x_1, x_2 \in R_+$ Therefore, f(x) is one-one function. For onto: $9x^2 + 6x - 5-y = 0$ $\Rightarrow x = \frac{-1 \pm \sqrt{y+6}}{3}$ As $x \in R^+$, so $y \geq -5$ i.e range= $[-5, \infty)$ = Co-domain. Hence f is onto.	2 0.5 2.5 2.5	
33	Given that, $A = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix}$, To find A^{-1}. $\lvert A \rvert = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix} = 1(-1-2) -2(-2)+0 =$ $-3+4=1 \neq 0$		Exemplar

	Hence ,A^{-1} exists. Let Cij represent the cofactor of (i,j)th Element of A. Then,	1	
	C_{11} = -3, C_{21} = -2 , C_{31} = -4 C_{12} = 2 , C_{22} = 1 , C_{32} = 2 C_{13} = 2 , C_{23} = 1 , C33 = 3	1	
	Adj. A = $\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	0.5	
	$A^{-1} = \frac{\text{Adj.A}}{\lvert A \rvert} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	0.5	
	The given system of equations is equivalent to the matrix equation $A^T X = B \Rightarrow X = (A^T)^{-1} B$ $\Rightarrow X = (A^{-1})^T B$	0.5	
	$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -30 + 16 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix}$ $= \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$, Hence, $x = 0, y = -5, and\ z = -3$	1.5	
	OR		
	$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	1	NCERT
	$\Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$	1	
	Now, given system of equations can be written,in matrix form, as follows	1	

	$\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\1\\2\end{bmatrix}$ $\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}^{-1}\begin{bmatrix}1\\1\\2\end{bmatrix}$ $\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix}=\begin{bmatrix}-2+0+2\\9+2-6\\6+1-4\end{bmatrix}=\begin{bmatrix}0\\5\\3\end{bmatrix}$ $\Rightarrow$x=0,y=5,z=3	1 1	
34	(LHD at x=1)= $\lim_{x\to1^-}\frac{f(x)-1}{x-1}$ = $\lim_{h\to0}\frac{h^2-5h}{-h}$ =5 Now, (RHD at x=1)= $\lim_{x\to1^+}\frac{f(x)-1}{x-1}$ =$\lim_{h\to0}\frac{bh}{h}$ =b Since, f(x) is differentiable, so (LHD at x=1)=(RHD at x=1) b=5 And f(1)=1+3+a=4+a Now, LHL= $\lim_{x\to1^-}$ f(x) LHL=$\lim_{h\to0}(1-h)^2+3(1-h)+a$ LHL=1+3+a=4+a Now, RHL= $\lim_{x\to1^+}$ f(x) RHL=$\lim_{h\to0}$ b(1+h) RHL=b+2 Since, f(x) is continuous, so LHL=RHL=f(1) 4+a=b+2 4+a=5+2 a=7−4=3 Hence, a=3 and b=5.	1 1 1 1 1	Exemplar
35	Prove of $\int_0^t f(x)\,dx = \int_0^t f(t-x)\,dx$ $\int_0^{\pi}\frac{\pi\tan x}{secx+tanx}dx = \int_0^{\pi}\frac{(\pi-x)\tan(\pi-x)}{sec(\pi-x)+tan(\pi-x)}dx$	2	NCERT

	$\int_0^{\pi} \frac{(\pi - x)\tan(x)}{sec(x) + tan(x)} dx$ $\int_0^{\pi} \frac{(\pi - x)\tan(x)}{sec(x) + tan(x)} dx$ 2 I = $\int_0^{\pi} \frac{(\pi)tanx}{sec(x)+tan(x)} dx$ $=[\sec x - \tan x + x]_0^{\pi}$ $=\frac{\pi}{2}(\pi - 2)$	1 1 1	
SECTION – E			
36	(i)Let A be the 2 × 3 matrix representing the annual sales of products in two markets. $\therefore A = \begin{bmatrix} x & y & z \\ 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}$ Let B be the column matrix representing the sale price of each unit of products x, y, z. $\therefore B = \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$ Now, revenue = sale price × number of items sold Therefore, the revenue collected from Market I = ₹ 46000. (ii) The revenue collected from Market II = ₹ 53000. (iii) Let C be the column matrix representing cost price of each unit of products x, y, z. Then, $C = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$ Total cost in each market is given by $AC = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$ Now, Profit matrix = Revenue matrix - Cost matrix = Therefore, the gross profit from both the markets = ₹ 15000 + ₹ 17000 = ₹ 32000 OR |A|=1000 |A| + |adj(A)|=1001000	1 1 2 1 1	NCERTI
37	Let length of square piece to be cut of be x mt. Length of box is = (8 - 2x) mt. Breadth = (3 – 2x) mt. Height = x unit (i) Volume of the box V = $x(3 - 2x)(8 - 2x)$	1	Ncert

	(ii) $\frac{dv}{dx} = (3-4x)(8-2x) + (3x-2x^2)(-2)$ $= 12x^2 - 44x + 24 = 4(3x^2 - 11x + 6)$ For $\frac{dv}{dx} = 0 \Rightarrow 3x^2 - 11x + 6 = 0$ $\Rightarrow x = 3 \ or\ x = 2/3$ $x = 3$ is not possible. So $x = 2/3$. The length of square piece is 2/3 mt. (iii) For $x < \frac{2}{3}$, $\frac{dv}{dx} > 0$ For $x > \frac{2}{3}$, $\frac{dv}{dx} < 0$ As $\frac{dv}{dx}$ changes sign from +ve to –ve as x increases So volume is maximum at $x = \frac{2}{3}$. Hence Max. Volume is 280/27 m^3. OR $\frac{d^2v}{dx^2} = 4(6x-4)$ $for\ x = \frac{2}{3}, \frac{d^2v}{dx^2} = 4\left(6 \times \frac{2}{3} - 11\right) = -28 < 0$ Volume is maximum at $x = \frac{2}{3}$. Hence Max. Volume is 280/27 m^3	1 2 2	
38	(i) Point of intersections are (0,2) and (3,0) Value of the given integral is 3/2 (ii) Required area= $\frac{3\pi}{2} - 1$	1 Mark 1 Mark 2 Marks	NCERT